Uncertainity Involved in Job - Shop Scheduling

Abstract

Theoretically as well as practically, a job shop scheduling problem is a difficult task. The difficulty is due to higher number of constraints, which are unavoiadable in the real world situations. This problem is a combinatiorial optimization of considerable industrial importance. This paper investigates the job-shop scheduling problem with imprecise processing time and use the fuzzy numbers to represent imprecise processing times. We first use triangular fuzzy numbers to represent imprecise processing times, and then construct a fuzzy job-shop scheduling model. The objective of this paper is to extend the deterministic job-shop scheduling problem into a more generalized problem that will be useful in many practical situation.

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t - processing time.
$\tilde{t}$, $\tilde{c}$ - level 1 triangular Fuzzy Number.
DM - Decision maker.

Introduction

The job-shop scheduling problem is concerned with allocating limited resources to operations over time. Between the operations precedence constraints for a job can be defined. Although job-shop scheduling has always had an important role in the field of production and operations management, it is a difficult problem in combinatorial optimization. The difficulty is due to the high number of constraints, unfortunately unavoidable in the real-world applications. The job-shop scheduling problem can be described as follows. We are given n jobs and m machines. Each job consists of a sequence of operations that must be processed on m machines in a given order. Each operation must be executed uninterrupted on a given machine for a given period of time and each machine can only handle at most one operation at a time. The problem is to find a schedule, an allocation of the operations of n jobs to certain time intervals on in machines, with a minimum overall time.

Objective of the Study

In this study, we investigate a job-shop scheduling problem with imprecise processing time and use fuzzy numbers to represent imprecise processing times in this problem. The main interest of our approach is that the fuzzy schedules obtained from (1-5)are the same type as those in the crisp job-shop scheduling problem. The fuzzy job-shop scheduling model, in the case of imprecise processing times, is then an extension of the crisp problem.

Job-Shop Scheduling Problem

The deterministic job-shop scheduling problem is stated as follows. There are n jobs to be scheduled on m machines. Each job consists of a sequence of operations that must be processed on m machines in a given order. Each operation is characterized by specifying both the required machine and the fixed processing time. Several constraints on jobs and machines, which are listed as follows:

1. Each job must pass through each machine once and only once.
2. Each job should be processed through the machine in a particular order.
3. Each operation must be executed uninterrupted on a given machine.
4. Each machine can only handle at most one operation at a time.

The problem is to find a schedule to determine the operation sequences on the machines in order to minimize the total completion time. Let c_{ik} denote the completion time of job i on machine k, and t_{ik} denote the processing time of job i on machine k. For a job i,if the processing on machine h precedes that on machine k, we need the following constraint:

$$c_{ik} - t_{ik} \geq c_{ih}$$

On the other hand, if the processing on machine k comes first, the constraint becomes

$$c_{ih} - t_{ih} \geq c_{ik}$$

Thus, we need to define an indicator variable x_{ihk} as follows:

$$x_{ihk} = \begin{cases} 1, \text{processing on machine } h \text{ precedes that on machine } k \text{ for job } i \\ 0, \text{otherwise} \end{cases}$$

We can then rewrite the above constraints as follows:

$c_{ik} - t_{ik} + L(1 - x_{ihx}) \geq c_{ih}$, i = 1, 2, …, n, h, k = 1, 2, …. m

Where, L is a large positive number. Consider two jobs, i and j , that are to be processed on machine k. If job i comes before job j , we need the following constraint :

$c_{jk} - c_{ik} \geq t_{jk}$

Otherwise, if job j comes first, the constraint becomes

$c_{ik} - c_{jk} \geq t_{ik}$

Therefore, we also need to define another indicator variable y_{ijk} as follows.

$$y_{ijk} = \begin{cases} 1, \; if \;\; job \; i \; precedes \;\; job \; j \; on \;\; machine \; k \\ 0, \; otherwise \end{cases}$$

We can then rewrite the above constraints as follows :

$c_{jk} - c_{ik} + L(1 - y_{ijk}) \geq t_{jk}$, i, j = 1, 2, …., n, k = 1, 2, …., m

The job-shop scheduling problem with a makespan objective can be formulated as follows

$$\min \max_{1 \leq k \leq m} [\max_{1 \leq i \leq n}[\max[\, c_{ik}\,]] \quad (1)$$

$st.\;\; c_{ik} - t_{ik} + L(1 - x_{ihk}) \geq c_{ih}$ I = 1, 2, …, n, h, k = 1, 2, …., m (2)

$c_{jk} - c_{ik} + L(1 - y_{ijk}) \geq t_{jk}$, i, j = 1, 2, …, n, k = 1, 2, …..m, (3)

$c_{ik} \geq 0$, i = 1, 2, …., n, k = 1, 2, …., m (4)

x_{ihk} = 0 or 1, i = 1, 2, ….., n, h, k = 1, 2, …., m. (5)

y_{ijk} = 0 or 1, i, j = 1, 2, …., n, k = 1,2, ……m,

Constructing a Fuzzy Job-Shop Scheduling Model.

Imprecise Processing Time

As in real life situations, some unexpected events may occur, resulting in small changes to the processing time of each job. Therefore, in many situations, the processing time can only be estimated as being within a certain interval. Because of this interval estimation feature, the representation of processing time for a job can be more realistically and naturally achieved through the use of a fuzzy number. As a result, decision-makers (DM) do not need to give a single precise number to represent the processing time of a job. We use t_{jk}, the processing time for job j on machine k, to denote the process time for each job in this paper. However, t_{jk} is just an estimate and its exact value is actually unknown.

A Fuzzy Job-Shop Sequencing Model

Consider the schedule for the job-shop problem is performed several times in practical situations. Obviously, the processing time for this schedule at different execution times is not necessarily the same. Therefore, an estimated processing time interval, i.e. $[t_{jk} — \Delta_{jk1},\; t_{jk} + \Delta_{jk2}]$ should be given to represent the possible range of values for the processing time. Thus the use of interval $[t_{jk} — \Delta_{jk1},\; t_{jk} + \Delta_{jk2}]$ is more appropriate than the use of a single estimate, t_{jk}, in practical situations. The DM should carefully determine the parameters Δ_{jk1} and Δ_{jk2}, which satisfy 0 < Δ_{jk1} < t_{jk}, 0< Δ_{jk2} for defining an acceptable processing time range for any particular problem. After that, the DM can choose an appropriate value from the interval $[t_{jk} — \Delta_{jk1},\; t_{jk} + \Delta_{jk2}]$ as an estimate for the processing time for the job j on machine k. Obviously, when the estimate happens to be t_{jk}, which is the crisp processing time, the error rate is zero. When the estimate deviates from t_{jk}, the error rate will become larger. In fact, we can use the term "confidence level" instead of "error rate" while we consider processing time interval based on the fuzzy viewpoint. We can therefore say that the confidence level is one if the processing time estimate equal to t_{jk}. Otherwise, when the processing time estimate deviates from t_{jk}, the confidence level will be smaller. Finally, when estimates approaches one of the two ends of the interval, i.e. t_{jk} - Δ_{jk1} or t_{jk} + Δ_{jk2}, the confidence level will be close to zero.

A level 1 triangular fuzzy number corresponding to the above interval $[t_{jk} — \Delta_{jk1},\; t_{jk} + \Delta_{jk2}]$ is given as follows :

$$\tilde{t}_{jk} = (t_{jk} - \Delta_{jk1}, t_{jk}, \ \ t_{jk} + \Delta_{jk2}; 1) \in F_N(1), \quad (6)$$

$0 < \Delta_{jk1} < t_{jk}$ -, $0 < \Delta_{jk2}$, = 1,2, ….. n,
k = 1, 2, …..m.

Fig. 1 : A triangular fuzzy number $\tilde{t}_{jk}$

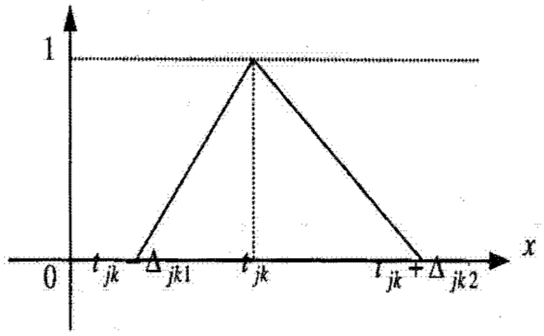


Fig. 1 shows a level 1 triangular fuzzy number $\tilde{t}_{jk}$. From fig. 1 we can see that the membership grade at t_{jk} in $\tilde{t}_{jk}$ is 1. However, the more the fuzzy number deviates from the t_{jk} position, the lesser the fuzzy numbers membership grade in $\tilde{t}_{jk}$. The membership grade at $t_{jk} - \Delta_{jk1}$ or $t_{jk} + \Delta_{jk2}$ is zero. Of course, we can see that the confidence level for an estimate in the interval corresponds to the membership grade of a fuzzy number in the fuzzy sets. This concept naturally leads to the use of fuzzy numbers for the processing time in the job-shop scheduling problem.

Let $\tilde{C}_{ik} = (c_{ik}, c_{ik}, c_{ik}; 1) = (\tilde{C}_{ik})_1 \in F_N(1)$. (7)

Now we define ranking on triangular fuzzy numbers, so that fuzzify (1) - (4) using (6) and (7) to obtain the fuzzy job-shop scheduling problem.

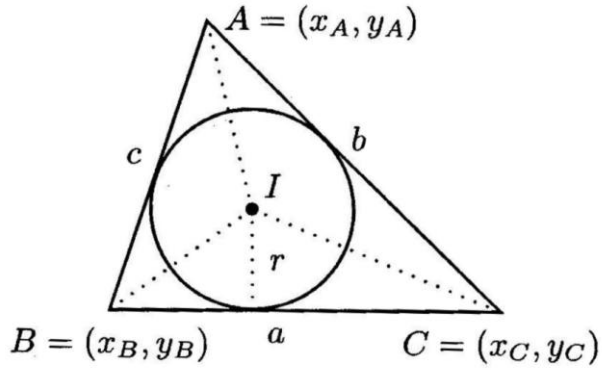
Here, we present a new method for ranking triangular fuzzy numbers which considers the incircle of a triangular fuzzy number. The incircle of a triangle is that circle which just touches all three sides of the triangle. The proposed method can also rank the crisp numbers and triangular fuzzy numbers with the same centroid point. The presented method is based on the uniqueness of the incircle of a triangle which is given by the following theorem:2

Theorem 1

The angle bisectors of a triangle intersect at a common point I called the incenter, which is the center of the unique circle inscribed in the triangle (called the incircle).

Let us suppose that three vertices of a triangle ∆ABC are located at (x_A, y_A), (x_B, y_B), and (x_C, y_C) and the opposite sides of the triangle have lengths a, b, and c respectively (see Fig. 2). Then Heron's formula tells us that the area of a triangle is determined by the lengths of its three sides.

Fig. 2 The Incircle, Incenter and Inradius of triangle ∆ABC



Theorem 2

The area $|\Delta ABC|$ of triangle ∆ABC in a Euclidean space R^n is given by Heron's formula

$$|\Delta ABC| = \sqrt{s(s-a)((s-b)(s-c)}, \quad ....(1)$$

Where, s = $\frac{a+b+c}{2}$ is the semiperimeter of the triangle.

We now give a lemma that allows us to derive a formula for the inradius of the incircle.

Lemma 1

The area $|\Delta ABC|$ of triangle ∆ABC in a Euclidean space R^n is equal to the product of its inradius *r* and semiperimeter *s.*

Using Theorem 2 and Lemma 1 we get formula (2) for the inradius of *r* of the incircle as follows :

$$|\Delta ABC| = rs \quad \Rightarrow \quad r = \frac{|\Delta ABC|}{s}$$

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s} \quad (2)$$

On the other hand, since the incenter *I* of the incircle is the point on the interior of the triangle that is equidistant from the triangle's three sides, it is not difficult to derive formula (3) for the incenter.

Theorem 3

The incenter *I* of triangle ∆ABC in a Euclidean space R^n satisfies the equation.

$$I = \frac{a(x_A, y_A) + b(x_B, y_B) + c(x_C, y_C)}{P} \quad (3)$$

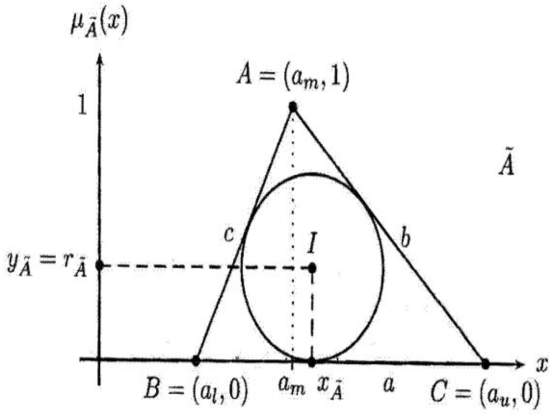
Here, P = a + b + c is perimeter of the triangle ∆ABC.

Now, let us use these results to rank triangular fuzzy numbers. Let $\tilde{A}$ = (a_l, a_m a_u) be a triangular fuzzy number (see Fig. 3). Then :
(x_A, y_A) = (a_m, 1), (x_B, y_B) = (a_l, 1),, (x_C, y_C) = (a_u, 0) and by the distance formula we get,
a = a_u - a_l,

b = $\sqrt{1+(a_u-a_m)^2}$, $c=\sqrt{1+(a_m-a_1)^2}$,

P = (a_u - a_1) + $\sqrt{1+(a_u-a_m)^2}$ + $\sqrt{1+(a_m-a_1)^2}$ (4)

Fig. 3 Triangular Fuzzy Number $\tilde{A}$ with Its Incircle, Incenter and Inradius.



Substituting (4) into (2) and (3), we obtain :

$$r_{\tilde{A}} = \frac{a_u-a_l}{(a_u-a_l)+\sqrt{1+(a_u-a_m)^2}+\sqrt{1+a_m-a_l)^2}}, \quad (5)$$

$$I_{\tilde{A}}(x_{\tilde{A}},y_{\tilde{A}}) = \frac{(a_u-a_l)(a_m,1)+\sqrt{1+(a_u-a_m)^2}\,(a_l,0)+\sqrt{1+(a_m-a_l)^2}\,(a_u,0)}{(a_u-a_l)+\sqrt{1+(a_u-a_m)^2}+\sqrt{1+(a_m-a_l)^2}} \quad (6)$$

Furthermore, since the base of the triangle lies on the x-axis, and the incircle is tangent to the x-axis, the ordinate of the incenter $y_{\tilde{A}}$ *< 1/2* (See Fig. 3).

Let $\tilde{A}$ = *(a_m - $\in_1$, a_m, a_m + $\in_2$)* be a triangular fuzzy number, where $\in_1, \in_2$ *are* arbitrary small positive real numbers. Then using (5) we get

$$\lim_{\in_1,\in_2\to 0^+} r_{\tilde{A}} = \lim_{\in_1,\in_2\to 0^+} \frac{\in_2+\in_1}{(\in_2+\in_1)+\sqrt{(1+\in_2^2}+\sqrt{(1+\in_1^2}} = 0$$

Similarly, using (6) we obtain

$$\lim_{\in_1,\in_2\to 0^+} x_{\tilde{A}} = \lim_{\in_1,\in_2\to 0^+} \frac{(\in_1+\in_2)\,a_m+(a_m-\in_1)\sqrt{1+\in_2^2}+(a_m+\in_2)\sqrt{1+\in_1^2}}{(\in_2+\in_1)+\sqrt{(1+\in_2^2}+\sqrt{(1+\in_1^2}}$$

$$= \frac{a_m}{2}$$

Since, $y_{\tilde{A}}$ = $r_{\tilde{A}}$ = we have $I_{\tilde{A}}$ ($x_{\tilde{A}}$, $y_{\tilde{A}}$) = $\left(\frac{a_m}{2},0\right)$. Therefore, if triangular fuzzy number $\tilde{A}$ *(a_l, a_m, a_u)* is a crisp number, that is, a_l = a_m = a_u then we can assume that $r_{\tilde{A}}$ = 0 and $I_{\tilde{A}}$ = (a_m, 0).

It is obvious from (5) and (6) that the incenter and inradius of a triangular fuzzy number continuously depend on the parameters al, am and au. Therefore, an immediate idea that comes to mind is to rank triangular fuzzy numbers with respect to their incircles. Using the notations given above, intuitively, and being consistent with results in the literature, we define the rank of the triangular fuzzy number $\tilde{A}$ = (a_l, a_m, a_u) as

$$\text{Rank } (\tilde{A}) = \left(x_{\tilde{A}} - \frac{1}{2} y_{\tilde{A}}, 1 - y_{\tilde{A}}, a_m \right) \quad (7)$$

That is, the rank of the triangular fuzzy number is a triplet which depends on its incenter and peak point.

The job-shop scheduling problem with imprecise processing time modeled by fuzzy number is as follows:

$$\min \max_{1\le k\le m} [\max_{1\le i\le n} [c_{ik}]] \quad (8)$$

s.t. $c_{ik} - t^*_{ik} + L(1-x_{ihk}) \ge c_{ih}$, $i=1,2,...,n$, $h,k=1,2,...,m$ (9)

$c_{jk} - c_{ik} + L(1-y_{ijk}) \ge t^*_{jk}$, $i,j=1,2,...,n$, $k=1,2,...,m$ (10)

$c_{ik} \ge 0$, $i=1,2,...,n$, $k=1,2,...,m$ (11)

$x_{ihk} = 0$ or 1, $i=1,2,...,n$, $h,k=1,2,...,m$ (12)

$y_{ijk} = 0$ or 1, $i,j=1,2,...,n$, $k=1,2,...,m$

Conclusion

The interpretation of Fig. 1 is as follows : When Δ_{jk2} > Δ_{jk1}, $\forall j,k$, the triangle is skewed to the right-hand side, thus obtaining t^*_{jk} > t_{jk} , $\forall j,k$. This means that the completion time in the fuzzy sense is longer than in the crisp case. Conversely, when Δ_{jk2} < Δ_{jk1}, $\forall j,k$, the triangle is skewed to the left-hand side, thus obtaining t^*_{jk} < t_{jk} , $\forall j,k$, indicating that the completion time in the fuzzy sense is shorter than the crisp case .

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